Post-Snowden Elliptic Curve Cryptography

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June 2013 – the Snowden leaks

The New York Times

"… the NSA had written the [crypto] standard and could break it."

Post-Snowden responses

- *Bruce Schneier: "I no longer trust the constants. I believe the NSA has manipulated them…"*
- *TLS WG makes formal request to CFRG for new elliptic curves for usage in TLS*
- *NIST announces plans to host workshop to discuss new elliptic curves*

Our motivations

1. **Curves that regain confidence and acceptance from public**

   - simple and rigid generation / “nothing up my sleeves”

2. **Improved performance and security for standard ECC algorithms and protocols**

   - new curve models

   - faster finite fields

   - side-channel resistance

Industry moving to Perfect Forward Secrecy (PFS) modes (e.g., ECDHE)

(e.g., see “Protecting Customer Data from Government Snooping” by Brad Smith, Microsoft General Counsel http://blogs.microsoft.com/blog/2013/12/04/protecting-customer-data-from-government-snooping/)

“Nothing-Up-My-Sleeve” (NUMS) curve generation

Case with Edwards form, $\boldsymbol{p = 3 (\mathrm{mod}\ 4)}$

Define the Edwards curve $E_d/\mathbb{F}_p: x^2 + y^2 = 1 + dx^2y^2$ with quadratic twist $E'_d/\mathbb{F}_p: x^2 + y^2 = 1 + (1/d)x^2y^2$.

1. Pick a prime p according to well-defined efficiency/security criteria
2. Find smallest $|d| > 0$, with d non-square in $\mathbb{F}_p$, such that $\#E_d = h \times r$ and $\#E'_d = h' \times r'$, where r, r' are primes and $h = h' = 4$

Note: for both Edwards and twisted Edwards, minimal d corresponds to minimal Montgomery constant $(A + 2)/4$ up to isogeny

“Nothing-Up-My-Sleeve” (NUMS) curve generation [1]

Case with twisted Edwards form, $\mathbf{p = 1 (mod\ 4)}$

Define the twisted Edwards curve $E_d/\mathbb{F}_p: -x^2 + y^2 = 1 + dx^2y^2$ with quadratic twist $E'_d/\mathbb{F}_p: -x^2 + y^2 = 1 + (1/d)x^2y^2$.

1. Pick a prime p according to well-defined efficiency/security criteria
2. Find smallest $|d| > 0$, with d non-square in $\mathbb{F}_p$, such that $\#E_d = h \times r$ and $\#E'_d = h' \times r'$, where r, r' are primes and $\{h, h'\} = \{4,8\}$ [2]

[1] The NUMS generation algorithm was presented in Bos et al. “Selecting Elliptic Curves for Cryptography: An Efficiency and Security Analysis”, http://eprint.iacr.org/2014/130, and extended to $p = 1(\mathrm{mod}\ 4)$ in Black et al., http://tools.ietf.org/html/draft-black-rpgecc-00.

[2] In addition, care must be taken to ensure MOV degree and CM discriminant requirements.

“Nothing-Up-My-Sleeve” (NUMS) curve generation

- It can be easily adapted to other curve forms.

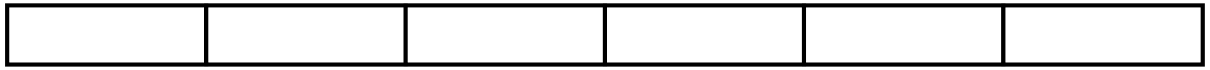
- There are several alternatives for primes: pseudo-random, pseudo-Mersenne, “Solinas” primes, etc.
  - Our original preference to balance rigidity, consistency and efficiency was to fix $p = 2^{2s} - c$, where c is the smallest integer s.t. $p \equiv 3 \bmod 4$ for $s \in \{256, 384, 512\}$.
  - Later extended to $p \equiv 1 \bmod 4$ to enable the use of complete twisted Edwards additions

But if efficiency is the main criteria:

How do we select primes?

Selecting primes: saturated vs. unsaturated arithmetic

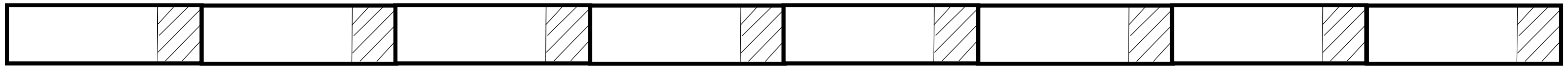
Saturated:



limbs = field bitlength/computer word bitlength

No room for accumulating intermediate values without word spilling

Unsaturated:



limbs $\geq \lceil(\text{field bitlength} + \delta)/\text{computer word bitlength}\rceil$, for some $\delta > 0$

Extra room for accumulating intermediate values without word spilling

Selecting primes: saturated vs. unsaturated arithmetic

Saturated:

- More efficient when operations with carries are efficient, multiplication is relatively expensive **(e.g., AMD, Intel Atom, Intel Quark, ARM w/o NEON, microcontrollers)**
- More amenable for “generic” libraries, support for multiple curves
- Cleaner/easier-to-maintain curve arithmetic

Unsaturated:

- More efficient when instructions with carries are relatively expensive **(e.g., Intel desktop/server)**
- More efficient when using vector instructions **(e.g., ARM with NEON)**
- (When using incomplete reduction) requires specialized curve arithmetic.
  Bound analysis is required: error prone, errors are more difficult to catch

Comparison of x64 implementations Unsaturated versus Saturated

Relative cost between Curve25519 amd64-51 (unsaturated) and amd64-64 (saturated). RED indicates amd64-64 is better

Intel Haswell (wintermute):	10%
Intel Ivy Bridge (hydra8):	6%
Intel Sandy Bridge (hydra7):	5%
Intel Atom (h8atom):	-36%
AMD Piledriver (hydra9):	-39%
AMD Bulldozer (hydra6):	-38%
AMD Bobcat (h4e450):	-47%

* Source: SUPERCOP, accessed 01/05/2015

A new high-security curve: Ted37919

Ted37919 is defined by the twisted Edwards curve

$$E\colon -x^2 + y^2 = 1 + 143305x^2y^2$$

defined over $\mathbb{F}_p$ with $p = 2^{379} - 19$. $\#E = 8r$, where $r = 2^{376} - 21264887305280274198387666383606401577591915095403210637 9$.

- Provides ~ 188 bits of security
- Minimal d in twisted Edwards form
- Minimal constant $(A + 2)/4$ in its isogenous Montgomery form
- Generated with the NUMS curve generation algorithm

➢ Implementation-friendly to both saturated and unsaturated arithmetic:
truly high efficiency *independent* of the platform for the 192-bit level

Comparison with other high-security curves

Number of limbs for the implementation of different fields (64 and 32-bit CPU)

Saturated arithmetic

$2^{379} - 19$ (Ted37919):	**6 64-bit limbs** or **12 32-bit limbs**
$2^{389} - 21$ (*):	7 64-bit limbs or 13 32-bit limbs
$2^{414} - 17$ (Curve41417):	7 64-bit limbs or 13 32-bit limbs
$2^{448} - 2^{224} - 1$ (Goldilocks):	7 64-bit limbs or 14 32-bit limbs

Unsaturated arithmetic

$2^{379} - 19$ (Ted37919):	**7 54/55-bit limbs** or **15 25/26-bit limbs**
$2^{389} - 21$ (*):	7 55/56-bit limbs or 15 25/26-bit limbs
$2^{414} - 17$ (Curve41417):	8 51/52-bit limbs or 16 25/26-bit limbs
$2^{448} - 2^{224} - 1$ (Goldilocks):	8 56-bit limbs or 16 28-bit limbs

(*) The use of this prime has been discussed on the CFRG mailing list
(e.g., see http://www.ietf.org/mail-archive/web/cfrg/current/msg05733.html)

Comparison with other high-security curves

Cycles to compute scalar multiplication (on “unsaturated-friendly” platforms)

Curve	bit security	Intel Sandy Bridge	Intel Haswell
Ted37919, $p = 2^{379} - 19$	187.8	494,000	410,000
Ed448-Goldilocks, $p = 2^{448} - 2^{224} - 1$ (*)	222.8	658,000	532,000
E-521, $p = 2^{521} - 1$	259.3	1,030,000	803,000

- Ted37919 implementation is very simple, no use of more complex algorithms such as Karatsuba.
- Pure C versions cost 558,000 and 467,000 cycles on Intel SB and Haswell, respectively.

(*) Source: SUPERCOP, accessed on 01/05/2015

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Q&A

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